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Ya. Drin, Yu. Ushenko, I. Drin, S. Drin

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ABOUT THE APPROXIMATE SOLUTIONS TO LINEAR AND NON-LINEAR PSEUDODIFFERENTIAL REACTION DIFFUSION EQUATIONS

Background. The concept of fractal is one of the main paradigms of modern theoretical and experimental physics, radiophysics and radar, and fractional calculus is the mathematical basis of fractal physics, geothermal energy and space electrodynamics. We investigate the solvability of the Cauchy problem for linear and nonlinear inhomogeneous pseudodifferential diffusion equations. The equation contains a fractional derivative of a Riemann–Liouville time variable defined by Caputo and a pseudodifferential operator that acts on spatial variables and is constructed in a homogeneous, non-negative homogeneous order, a non-smooth character at the origin, smooth enough outside. The heterogeneity of the equation depends on the temporal and spatial variables and permits the Laplace transform of the temporal variable. The initial condition contains a restricted function.

Objective. To show that the homotopy perturbation transform method (HPTM) is easily applied to linear and nonlinear inhomogeneous pseudodifferential diffusion equations. To prove the solvability and obtain the solution formula for the Cauchy problem series for the given linear and nonlinear diffusion equations.

Methods. The problem is solved by the NPTM method, which combines a Laplace transform with a time variable and a homotopy perturbation method (HPM). After the Laplace transform, we obtain an integral equation which is solved as a series by degrees of the entered parameter with unknown coefficients. Substituting the input formula for the solution into the integral equation, we equate the expressions to equal parameter degrees and obtain formulas for unknown coefficients. When solving the nonlinear equation, we use a special polynomial which is included in the decomposition coefficients of the nonlinear function and allows the homotopy perturbation method to be applied as well for nonlinear non-uniform pseudodifferential diffusion equation.

Results. The result is a solution of the Cauchy problem for the investigated diffusion equation, which is represented as a series of terms whose functions are found from the parametric series.

Conclusions. In this paper we first prove the solvability and obtain the formula for solving the Cauchy problem as a series for linear and nonlinear inhomogeneous pseudodifferential equations.

Keywords: Laplace transform, Homotopy perturbation transform method, fractional reaction-diffusion equation, Caputo time-fractional derivative, pseudodifferential operator.

Introduction

The concept of fractal is one of fundamental paradigms of modern theoretical and experimental physics, radio physics and radar, and fractional calculus is the mathematical basis of fractal physics, geothermal energy and cosmic electrodynamics.

In recent years, fractional reaction-diffusion models are studied due to their usefulness and importance in many areas of science and engineering. The reaction-diffusion equations arise naturally as description models of many evaluation problems in the real world, such as the chemistry [1, 2], biology [3], finance [4-6] and hydrology [7]. Burke at [8] obtained solutions for enzyme-suicide substrate reaction with an instantaneous point source of substrate. In 1993 Grimson and Barker [9] introduced a continuum model for the spatio-temporal growth of bacterial colonies on the surface of a solid substrate with utilizes a reaction-diffusion equation for growth. Many cellular and sub-cellular biological processes [10] can be described in terms of diffusing and chemically reacting species (e.g. enzymes). A traditional approach to the mathematical modelling of such reaction-diffusion processes is to describe each biochemical species by its (spatially depend) concentration. Recently, interest in fractional reaction-diffusion equation [11-17] has increased because the equation exhibits self-organization phenomena and introduces a new parameter, the fractional index into the equation. Additionally, the analysis of fractional reaction-diffusion equations is of great importance from the analytical and numerical point of view. In [18] the authors obtain the analytical solutions of linear and nonlinear space-time fractional reaction-diffusion equations on a finite domain by the application of homotopy perturbation transform method. Numerical results show that the HPTM is easy to implement and accurate when applied to linear and non-linear space-time fractional reaction-diffusion equations.

The Riemann-Liouville fractional integral of order α is defined as [18, p.42, 19]

$$J^a_\alpha f(x,t) = \frac{1}{\Gamma(\alpha)} \int\limits_a^x (x-u)^{\alpha-1} f(u,t)du,$$
$$x > a. \tag{1}$$

The following fractional derivative of order $\alpha > > 0$ is introduced by Caputo [20]; see also Kilbas at all [21] in the form

$$ {}_aD_t f(x,t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int\limits_a^t \frac{f^{(m)}(x,\tau)d\tau}{(t-\tau)^{\alpha-1-m}}, \\ m-1<\alpha\le m, \\ \mathrm{Re}(\alpha)>0, \\ m\in\mathbb{N} \\ \dfrac{\partial^m f(x,t)}{\partial t^m}, \alpha = m, \end{cases} \tag{2}$$

where $\dfrac{\partial^m f(x,t)}{\partial t^m}$ is the m-partial derivative of$f(x,t)$ with respect to t.

The Laplace transform of the Caputo derivative is given by Caputo [20]; see also [21] in form

$$L\{{}_0D^\alpha_t f(x,t)\} = s^\alpha L[f(x,t)] -$$
$$-\sum_{r=0}^{n-1} s^{\alpha-r-1} f^{(r)}(x,0+), \quad (m-1<\alpha\le m). \tag{3}$$

The pseudodifferential operator A this symbol $a(\xi)$, $\xi\in\mathbb{R}^n$, $a(\nu\xi) = \nu^\beta a(\xi)$, $\nu>0$, is differentiable when $\xi\ne 0$, is defined as

$$Au(t,x) = F^{-1}_{\xi\to x}[a(\xi)F_{x\to\xi}[u(t,x)]],$$
$$t>0, x\in\mathbb{R}^n, \tag{4}$$

and $F_{x\to\xi}[u(t,x)] = v(t,\xi)$, $\xi\in\mathbb{R}^n$, $t>0$, $F^{-1}_{\xi\to x}[v(t,\xi)] = u(t,x)$ are direct and inverse Fourier transforms respectively [25].

Then $a(\xi) = |\xi|^\alpha(1+|\xi|^2)^{\gamma/2}$ in [22], $0<\alpha\le 2$, $\gamma>0$, is proof a formula for classical solutions for time- and space-fractional kinetic equation (also known as fractional diffusion equation) and deviation time variable is given in terms of the Fox's H-function, using the step by step method. These equations describe fractal properties of real data arising in applied fields such as turbulence, hydrology, ecology, geophysics, air pollution, economics and finance.

HPTM solutions of linear space-time fractional reaction-diffusion equation

First we consider the Cauchy problem for the linear space-time fractional reaction-diffusion equation in form

$${}_0D^\alpha_t u(x,t) = b(x)Au(x,t) - c(x)u(x,t) + f(x,t),$$
$$x\in\mathbb{R}^n, t>0, 0<\alpha\le 1, \tag{5}$$
$$u(x,0) = p(x), \quad x\in\mathbb{R}^n, \tag{6}$$

there operators is defined in (1) – (4), b, c, f, p is known functions [18].

Definition 1. Let $0<\alpha\le 2$. Suppose $u_0 \in \in C([0,\infty)\times\mathbb{R}^n)$, $f\in C([0,\infty)]\times R^n; C([0,\infty)\times \times\mathbb{R}^n))$. Then function $u\in C([0\infty)\times\mathbb{R}^n)$ is a classical solution of the Cauchy problem (1), (2), if

1) $F^{-1}_{\xi\to x}[(a(\xi))]$, $F_{x\to\xi}[u(t,x)]$ defines a continuous function of $x\in\mathbb{R}^n$ for each $t>0$ meant as hypersingular integral [25];

2) for every $x\in\mathbb{R}^n$, the fractional integral, as defined in (1), is continuously differentiable with respect to $t>0$, and $0<\alpha\le 2$.

3) the function $u(t,x)$ satisfies the integro-partial differential equation of (1) for every $(t,x)\in \in(0,\infty)\times\mathbb{R}^n$ and the initial condition (2) for every $x\in\mathbb{R}^n$.

Taking the Laplace transform on both sides of (5) and using (6) we get

$$s^\alpha Lu(x,t) - s^{\alpha-1}p(x) =$$
$$= b(x)LAu(x,t) - c(x)Lu(x,t) + Lf(x,t)$$

and

$$Lu(x,t) = \frac{p(x)}{s} + \frac{1}{s^\alpha}b(x)LAu(x,t) - \frac{c(x)}{s^\alpha}Lu +$$
$$+\frac{1}{s^\alpha}[Lf(x,t)](x,p). \tag{7}$$

Applying the inverse Laplace transform on both sides of (7) we get and

$$u(x,t) = L^{-1}\Big[\frac{p(x)}{s} + \frac{1}{s^\alpha}b(x)LAu(x,t) -$$
$$-\frac{c(x)}{s^\alpha}Lu(x,t) + \frac{1}{s^\alpha}Lf(x,t)\Big] = p(x) +$$
$$+L^{-1}\Big[\frac{1}{s^\alpha}b(x)LAu(x,t) - \frac{c(x)}{s^\alpha}Lu(x,t)\Big] +$$
$$+L^{-1}\Big[\frac{1}{s^\alpha}Lf(x,t)\Big]. \tag{8}$$

Now we apply the homotopy perturbation method, that is we are looking for solution of integral equation (8) as a poser series

$$u(x,t,p)=\sum_{n=0}^{\infty}p^n u_n(x,t), \tag{9}$$

where $u_n(x,t)$ are unknown functions, $p>0$ is parameter.

Substituting (9) in (8) we get

$$\sum_{n=0}^{\infty}p^n u_n(x,t)=p(x)+$$

$$+p\Big(L^{-1}_{s\to t}\Big[\frac{b(x)}{s^\alpha}L_{t\to s}\Big[A\sum_{n=0}^{\infty}p^n u_n(x,t)\Big]-$$

$$-\frac{c(x)}{s^\alpha}L_{t\to s}\Big[\sum_{n=0}^{\infty}p^n u_n(x,t)\Big]\Big]\Big)+$$

$$+L^{-1}\Big[\frac{1}{s^\alpha}L[f(x,t)]\Big].$$

Comparing the coefficients of the like terms of p, we have

$n=0$:

$$u_0(x,t)=p(x)+L^{-1}\Big[\frac{1}{s^\alpha}L_{t\to s}[f(x,t)]\Big]\equiv$$

$$\equiv p(x)+\hat{f}(x,t),$$

$n=1$:

$$u_1(x,t)=L^{-1}_{s\to t}\Big(\frac{b(x)}{s^\alpha}L_{t\to s}[Au_0(x,t)]-$$

$$-\frac{c(x)}{s^\alpha}L_{t\to s}[u_0(x,t)]\Big)=L^{-1}_{s\to t}\Big(\frac{b(x)}{s^\alpha}L_{t\to s}[A[p(x)+$$

$$+\hat{f}(x,t)]-\frac{c(x)}{s^\alpha}L_{t\to s}[p(x)+\hat{f}(x,t)]\Big),$$

where

$$L^{-1}_{s\to t}\Big(\frac{b(x)}{s^\alpha}L_{t\to s}[Ap(x)]\Big)=b(x)Ap(x)\frac{t^\alpha}{\Gamma(\alpha+1)};$$

$$L^{-1}_{s\to t}\Big(\frac{b(x)}{s^\alpha}L_{t\to s}[A\hat{f}]\Big)=b(x)A(\hat{f})^2,$$

$$(\hat{f})^2=\widehat{(\hat{f})};$$

$$L^{-1}_{s\to t}\Big(\frac{c(x)}{s^\alpha}L_{t\to s}[Ap(x)]\Big)=c(x)p(x)\frac{t^\alpha}{\Gamma(\alpha+1)};$$

$$L^{-1}_{s\to t}\Big(\frac{c(x)}{s^\alpha}L_{t\to s}[\hat{f}(x,t)]\Big)=c(x)(\hat{f})^2.$$

So we get

$$u_1(x,t)=(b(x)A-c(x))\Big(p(x)\frac{t^\alpha}{\Gamma(\alpha+1)}+(\hat{f})^2\Big).$$

$n=2$:

$$u_2(x,t)=L^{-1}_{s\to t}\Big(\frac{b(x)}{s^\alpha}L_{t\to s}[Au_1(x,t)]-$$

$$-\frac{c(x)}{s^\alpha}L_{t\to s}[u_1(x,t)]\Big)=$$

$$=b(x)A\Big\{L^{-1}_{s\to t}\Big[\frac{1}{s^\alpha}L_{t\to s}[u_1(x,t)]\Big]\Big\}-$$

$$-c(x)L^{-1}_{s\to t}\Big[\frac{1}{s^\alpha}L_{t\to s}[u_1(x,t)]\Big]=$$

$$=(b(x)A-c(x))^2\Big\{p(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+(\hat{f})^3\Big\}.$$

Similarly

$$u_3(x,t)=(b(x)A-c(x))^3\Big\{p(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}+(\hat{f})^3\Big\}$$

and with help of mathematical induction we get

$$u_n(x,t)=(b(x)A-c(x))^n\Big\{p(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}+(\hat{f})^n\Big\},$$

and so on, in this manner, the rest of component of the homotopy perturbation series can be obtained. Thus the solutions in series form is given by

$$u(x,t)=p(x)+\hat{f}(x)+[b(x)A-$$

$$-c(x)]\Big[p(x)\frac{t^\alpha}{\Gamma(\alpha+1)}+\hat{f}^2(x,t)\Big]+\Big[b(x)A-$$

$$-c(x)\Big]^n\Big(p(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}+\hat{f}^{n+1}(x,t)\Big)+\ldots. \tag{10}$$

HPTM solutions of non-linear space-time fractional reaction-diffusion equation

Now we consider the non-linear space-time fractional reaction-diffusion equation of the form:

$${}_0D^\alpha_t u(x,t)=bAu(x,t)+f(u(x,t))+g(x,t),$$

$$t>0, x\in\mathbb{R}^n, 0<\alpha\le 1, 1<\beta\le 2, \tag{11}$$

$$u(x,0)=p(x),\quad x\in\mathbb{R}^n. \tag{12}$$

Definition 2. The classical solution of Cauchy problem (11), (12) formulated analogue as definition 1.

Taking the Laplace transform on both sides of (11) and using (12), we get

$$L_{t\to s}u(x,t)=\frac{p(x)}{s}+\frac{b(x)}{s^\alpha}L_{t\to s}[Au(x,t)]+$$

$$+\frac{1}{s^\alpha}L_{t\to s}[f(u(x,t))]+\frac{1}{s^\alpha}L_{t\to s}[g(x,t)]. \tag{13}$$

Applying the inverse Laplace transform on both sides of (13) we get

$$u(x,t)=p(x)+L^{-1}_{s\to t}\Big[\frac{b(x)}{s^\alpha}L_{t\to s}[Au(x,t)]+$$

$$+\frac{1}{s^\alpha}L_{t\to s}[f(u(x,t))]\Big] + J_t^\alpha g(x,t). \quad (14)$$

Now we apply the homotopy perturbation method

$$u(x,t)=\sum_{n=0}^{\infty}p^n u_n(x,t) \quad (15)$$

and the nonlinear term can be decomposed as

$$f(u(x,t))=\sum_{n=0}^{\infty}p^n H_n(u) \quad (16)$$

for some He's polynomicals $H_m(u)$ [23, 24] that are given by

$$H_n(u_0,u_1,\ldots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}\Big[f\Big(\sum_{i=0}^{\infty}p^i u_i(x,t)\Big)\Big],$$
$$n=0,1,\ldots. \quad (17)$$

Substituting (15) and (16) in (14) we get

$$\sum_{n=0}^{\infty}p^n u_n(x,t)=p(x)+$$
$$+pL_{s\to t}^{-1}\Big[\frac{b(x)}{s^\alpha}L_{t\to s}[A\sum_{n=0}^{\infty}p^n u_n(x,t)]\Big]+$$
$$+\frac{1}{s^\alpha}L_{t\to s}\Big[\sum_{n=0}^{\infty}p^n H_n(u(x,t))\Big]+J_t^\alpha g(x,t). \quad (18)$$

Comparing in (18) the coefficients of the like terms of p we have

$$u_0(x,t)=p(x)+J_t^\alpha g(x,t),$$

$$u_1(x,t)=L_{s\to t}L\frac{1}{s^\alpha}L_{t\to s}\Big[Au_0(x,t)+$$
$$+\frac{1}{s^\alpha}L_{t\to s}[H_0]\Big]=J_t^\alpha[bAu_0(x,t)]+J_t^\alpha[H_0]$$

and roceeding in a similar manner, we get

$$u_2(x,t)=J_t^{\alpha[bAu_1(x,t)]}+J_t^\alpha[H_1],$$

$$u_3(x,t)=J_t^{\alpha[bAu_2(x,t)]}+J_t^\alpha[H_2],$$

$$u_{n+1}(x,t)=J_t^{\alpha[bAu_n(x,t)]}+J_t^\alpha[H_n],$$

and so on, in this manner, the rest of the components of the homotopy perturbation series can be obtained. Therefore the solution in series form is given by

$$u(t,x)=p(x)+J_t^\alpha g(x,t)+J_t^\alpha[bUa_0(x,)]+J_t^\alpha[H_0]+$$
$$+J_t^\alpha[bAu_1(x,t)]+J_t^\alpha[H_1]+\cdots+$$
$$+J_t^\alpha[bAu_n(x,t)]+J_t^\alpha[H_n]+\ldots \quad (19)$$

where H_n is defined by (17).

Main theorem

The solutions of problems (5), (6) and (11), (12) are defined by (10) and (19) respectively.

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Дрінь Я. М., Ушенко Ю. О., Дрінь І. І., Дрінь С. С.

ПРО НАБЛИЖЕНІ РОЗВ'ЯЗКИ ЛІНІЙНИХ ТА НЕЛІНІЙНИХ ПСЕВДОДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ

Поняття фрактала є однією з основних парадигм сучасної теоретичної та експериментальної фізики, радіофізики та радіолокації, а дробове числення є математичною основою фрактальної фізики, геотермальної енергії та космічної електродинаміки та інших. Ми досліджуємо розв'язність задачі Коші для лінійних та нелінійних неоднорідних псевдодиференціальних рівнянь дифузії. Рівняння містить дробову похідну за часовою змінною типу Рімана–Ліувілля, визначену Капуто, та псевдодиференціальний оператор, який діє за просторовими змінними і побудований за однорідним, невід'ємного порядку однорідності, негладким у початку координат символом, достатньо гладким за межами початку координат. Неоднорідність рівняння залежить від часової і просторових змінних та допускає перетворення Лапласа за часовою змінною. Початкова умова містить обмежену функцію.

Мета: показати, що метод гомотопічної пертурбації HPTM (homotopy perturbation transform method) легко застосовувати до лінійних та нелінійних неоднорідних псевдодиференціальних рівнянь дифузії. Довести розв'язності та отримання формули для розв'язку у вигляді ряду задачі Коші для вказаних лінійних та нелінійних рівнянь дифузії.

Методи. Задача розв'язується методом HPTM, який поєднує перетворення Лапласа (Laplace transform) за часовою змінною і метод гомотопічної пертурбації (HPM – homotopy perturbation method). Після перетворення Лапласа отримуємо інтегральне рівняння, розв'язок якого шукаємо у вигляді ряду за степенями введеного параметра з невідомими коефіцієнтами. Після підстановки введеної формули для розв'язку у інтегральне рівняння прирівнюємо вирази біля однакових степенів параметра і отримуємо формули для невідомих коефіцієнтів. При розв'язуванні нелінійного рівняння використовується спеціальний поліноміал, який входить в коефіцієнти розкладу нелінійної функції і дозволяє застосувати метод гомотопічної пертурбації і для нелінійного неоднорідного псевдодиференціального рівняння дифузії.

Результатом є розв'язок задачі Коші для досліджуваного рівняння дифузії, який подається у вигляді ряду, членами якого є знайдені функції з параметричного ряду.

В цій праці вперше доведена розв'язність та отримана формула для розв'язку задачі Коші у вигляді ряду для лінійних та нелінійних неоднорідних псевдодиференціальних рівнянь дифузії.

Ключові слова: перетворення Лапласа, гомотопічний пертурбаційний метод, фрактал, дробова похідна за Капуто, псевдодиференціальний оператор.

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